

Ptolemy's Digression: Astrology's Aspects and Musical Intervals

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Abstract

The rationale behind aspects has been relatively unexamined in astrology's tradition, although much development has occurred over the centuries. This article explores a passage in Ptolemy's *Tetrabiblos* Book I, Chapter 14. While presenting the different aspect relationships, Ptolemy alludes to musical intervals, in addition to arithmetical relationships. Examining this and similar passages in Ptolemy's *Harmonics* and Plato's *Timaeus*, this article asserts the importance of specific harmonizing musical intervals to bring together planets, solve the ancient problem of action at a distance, and account for astrology's aspects.

Statement of the Problem

This essay addresses a problem in the development and continuity of astrology: how do astrologers, past and present, account for the astrological aspects? Aspects are the means by which a planet or position (such as the Ascendant or Lot of Fortune) makes contact with another planet or planets. Once an astrologer has designated a planet or position to answer a question posed to the astrological chart, aspects to the designated position provide information to help form an answer. Aspects are an important technique for answering questions and determining astrological outcomes.

How does this connection by aspect occur? Because aspects are based on the distances between two positions along the zodiac, the solution is not obvious.

Of course, one needn't question the rationale behind aspects at all. The view of aspects in Indian astrology is very straightforward. All aspects are cast forward in the zodiac, and each planet aspects the house (and

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sign) opposite to it. In addition, Mars aspects the fourth and eighth houses from itself, Jupiter aspects the fifth and ninth, and Saturn aspects the third and tenth from itself.¹ These rules are part of the astrological craft passed down within their tradition. Most ancient western astrologers also used aspects without questioning them, and were not different from their Indian cousins in this regard.

However, the Hellenistic mind, exemplified by Ptolemy in Book I of his *Tetrabiblos*, sought to give astrology a coherent theoretical form, and integrate astrology more completely with other fields of understanding. Because of the enduring influence of the *Tetrabiblos*, and the regard in which Ptolemy was held during the subsequent history of astrology (even to the present day), his views require close inspection.

In particular, we shall examine *Tetrabiblos* I, 14.² Here Ptolemy gives two accounts for aspects. The first one, the subject of this paper, is based on what we might call 'fractions' and 'super-fractions', μόρια and ἐπιμόρια. The second argument of Chapter 14 depicts sympathetic and unsympathetic aspects through the nature of the zodiacal signs (ζώδια) brought into aspect.

Ptolemy's first account in Chapter 14, although of a different style than the surrounding material, gives a provocative and critical account of aspects – one related to harmonics and the diatonic musical scale in particular. Later astrologers, from the Renaissance into the modern era, emphasised arithmetical and geometrical templates to account for aspects. Subordinating the musical component of aspects to arithmetical and geometrical ones has allowed some of the theoretical foundation to crumble: it is musical harmonies, not simply properties and combinations of numbers that account for the astrological aspects.

When I first learned about aspects as a new astrology student, the account given was wholly arithmetical and geometrical: because we can divide the whole circle by halves, quarters, thirds, and sixths, we can

¹ See J. Braha, *Ancient Hindu Astrology for the Modern Astrologer* (Hollywood, FL, 1986), p. 55.

² Using the numbering of the Boer-Boll (*Claudii Ptolemaei opera quae exstant omnia*, Vol. III, 1, *Apotelesmatica*, Leipzig, 1940, repr. 1957), Hübner (*Claudii Ptolemaei opera quae exstant omnia*, Vol. III, 1, 'Αποτελεσματικά, Stuttgart/Leipzig, 1998) and Schmidt (Claudius Ptolemy, *Tetrabiblos Book I*, trans. R. Schmidt [Berkeley Springs, WV, 1994]) editions. The Robbins translation (*Ptolemy: Tetrabiblos*, F. E. Robbins, trans., Cambridge, MA, 1940, repr. 1994) has this as Chapter 13.

connect planets to one another by aspect. These relationships also give us the line, square, and what astrologers call the trine and sextile,³ geometrically the sides of a triangle and hexagon inscribed within a circle. These aspects divide the whole circle into twelfths. We might also ask, however, why it is that we do not transform the dodecahedron, a twelve-sided figure, and thus a side of 30°, into an aspect as well? It should fit conveniently with the others, but the 30° interval was not considered a true aspect in the ancient tradition, nor is it considered one by most modern astrologers.

What is it about number relationships that empower planets to act upon one another, in spite of their distance from each other? Paradoxically, aspecting planets act upon one another *because* of their distance from each other.

The modern mind has an easier time comprehending action at a distance because the physics of the modern era has made it possible for us to imagine it. If we are not of a theoretical bent, we have our various remotes to unlock our cars, open the garage door, control our televisions and radios, and so on. Because of our background in popular science and technology, modern astrologers tend not to raise a skeptical eyebrow to the understanding and use of astrological aspects. For the ancients, however, this does not solve the problem of ‘action at a distance’, since they had neither gravity nor electro-magnetic waves to account for the effect of aspects. Most of the Greek words for aspects are those of seeing or looking. Directly or indirectly, aspects are acts of visual perception.

Ancient astrologers thought of aspects in terms of seeing and being seen. A planet ‘looks ahead’ – ἐπιθεωρέω (*epitheōreō*) – to another planet, forward in the zodiac, to which it is in aspect. In return, the aspected planet ‘casts rays’ – ἀκτινοβολέω (*aktinoboleō*) – back to the aspecting planet.⁴ In return, an aspecting planet may ‘testify to’ or ‘witness’ – ἐπιμαρτυρέω (*epimartureō*) – another planet. Our English ‘aspect’ is also a seeing word, as is the Sanskrit word for aspect, *drishti*, a ‘gaze’ or ‘glance’. Two planets in the same sign (ζῳδιον) – the modern ‘conjunction’ – are not in aspect. Seeing words are not used for this relationship; instead, planets in the same sign are considered to be *with*

³ These are angles of 120 and 60 degrees respectively.

⁴ Among many other examples, see Hephaestio of Thebes, *Apotelesmatics Book I*, trans. R. Schmidt (Berkeley Springs, WV, 1994), Chapter 16; Antiochus of Athens, *The Thesaurus*, trans. R. Schmidt (Berkeley Springs, WV, 1993), Chapter 20.

one another. A planet must be outside its own immediate zodiacal environment in order to see or be seen.

Modern astrologers might say that vision is action at a distance, since we routinely see things distant from us. Our science tells us that light waves are a visible segment of a vast vibratory spectrum that surrounds us. These waves provide a bridge between us and an observed object, although an otter or a bat might 'see' something quite different. Ancient tradition gives a variety of accounts for visual perception. To our modern sensibilities, they range from the relatively straightforward to the very strange.

Aristotle's *De Anima* regards touch rather than vision as the most basic – and paradigmatic – sense faculty, and posits that vision, like the other sense faculties, has a medium (μεταξύ) by which the object carries itself to the perceiving subject. For vision, this medium is *light*. The object alters the light by which the view of the object comes to the subject: 'For what is to be colour is, as we say, just this, that it is capable of exciting change in the operantly (actual) transparent medium: and the actuality of the transparent is light' (418b).⁵

According to the Stoics, what binds together the object of perception and the subject is the tensing of *pneuma*. In the case of seeing, 'the seeing-*pneuma* in the eye makes the object visible by "tensing" the air-*pneuma* into a kind of illuminated cone with the object at base and eye at apex; the tension of this air is experienced as sight.'⁶

The Epicurean school posited images flowing from the objects themselves in a constant manner that the eye picks up.⁷

Another possibility, of uncertain seriousness, is found in Plato's *Timaeus* (45b-d). After commenting on the fact that the human body is well suited for the faculty of sight, especially to look up toward the heavens, Plato notes that vision occurs by means of the fire of daylight, another kind of fire in one's eye faculty and fire emanating from the observed object. As they connect, we see something.⁸

⁵ Translated by R. D. Hicks, in *Aristotle's De Anima in Focus*, ed. M. Durrant (London, 1993), p. 36.

⁶ J. Annas, *Hellenistic Philosophy of Mind* (Berkeley, CA, 1992), p. 72.

⁷ *Ibid.*, pp. 158-59.

⁸ F. Cornford, *Plato's Cosmology* (Indianapolis, 1935/1997) [hereafter *Plato's Cosmology*], p. 152.

When discussing how various life factors are described from the natal chart, Ptolemy also uses words for seeing and looking (as well as witnessing or testifying) when referring to aspects. However, he does *not* use these words in *Tetrabiblos* I when giving an account of the aspects themselves. Since he did not use visual perception to account for the rationale behind aspects, the usual ancient theories about the workings of vision are not helpful to us here. If the act of looking or seeing requires a medium to connect the object of sight with the subject, it is not at all clear what medium could transmit the aspects of astrology. The medium must be the aspect intervals themselves, but how?

Outside and Inside Ptolemy's Digression

Before we discuss the harmonic and musical material in I, 14, we need to place this material in context. Preceding and following Ptolemy's account of aspects in Chapter 14 is material solely related to the operation of the signs of the zodiac as discrete units.

Chapter 12 discusses signs (which Ptolemy here calls 'twelfth-parts') as cardinal (related to a solstice or an equinox), fixed and mutable (double-bodied). Chapter 13 classifies them as masculine or feminine. The end of chapter 14, continuing the topic of aspects, states that trines and sextiles are harmonious (σύμφωνοι) because their genders agree, and squares are inharmonious (ἀσύμφωνοι) because their genders differ.

Chapter 15 designates signs as commanding and obeying, based respectively on their northern or southern declination, symmetrical to the 0° Aries/0° Libra axis. (These are symmetrical with respect to their rising times.) Chapter 16 concerns itself with the signs of 'equal power', symmetrical to the 0° Cancer/0° Capricorn axis, and spending equal amounts of time above the horizon. Chapter 17 takes up aspects again and tells us that zodiacal signs are averse (ἀσύνδετα) when they are not familiar by aspect relationship, nor in a relationship of commanding/obeying or equal power, i.e. symmetrical to the cardinal axes. Chapters 18-20 discuss the affiliations of the zodiacal signs to planets by means of domicile, triplicity and exaltation. Up to this point, Ptolemy has been considering signs only as indivisible units. He deviates from this position in his first account of the astrological aspects, the topic of this paper.

Now let us return to the first part of Chapter 14. Although Books III and IV of the *Tetrabiblos* use conventional terminology for aspects based on vision, in Chapter 14 Ptolemy uses the words σχηματίζω (arrange in a figure) and συσχηματίζω (configure) for aspects. These words refer to

forming a figure or a posture, as a group of dancers in an ensemble performance. An example of this can be seen in Plotinus, *Enneads* IV, 33.11-16, where the 'figures' and 'configuration' of a dance are described using the related nouns σχῆμα and σχηματισμός.

This is a rather striking metaphor. It appears that, instead of describing aspects in the conventional sense of planets looking forward and back from each other, Ptolemy alludes to *our* perceptions when watching the planets arranged with one another. Anyone who observes the planets at night over time admires their interweaving movement, similar to the performers of a slowly moving ritual dance. Ptolemy reminds us here that aspects are not just geometrical arrangements between static points but meaningful moments captured within the larger patterns of planetary motion.

Ptolemy talks about the aspects as consisting of certain numbers of degrees, rather than as relationships between signs. Many modern astrologers, including me, at first, read this passage uncritically, as if there was nothing unusual happening here. Yet in the context of his emphasis on the fact that aspects for planets are made between *whole signs* in this part of *Tetrabiblos* I, the degree numbers seem out of place.

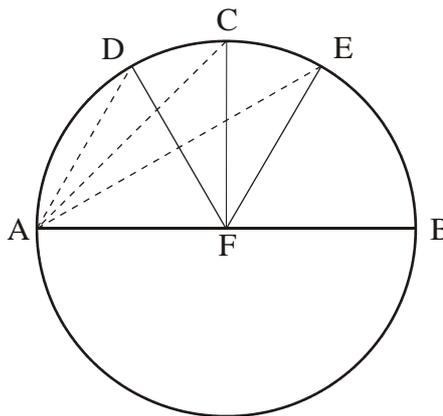
Using aspects from sign to sign, from one planet in Aries to another in Gemini, for example, planets could be in aspect to each other regardless of where in their respective signs they happen to be. Two planets in sextile by whole signs could be distant from each other anywhere from 31° (late Aries to early Gemini) to 89° (early Aries to late Gemini). As long as these planets are two signs from each other, they are in a hexagonal interval, i.e. in sextile. Why, then, does Ptolemy specify 180° for the diameter or opposition, 90° for a square, 60° for a hexagon, and 120° for a triangle? To derive these aspects, Ptolemy employs fractions (μόρια) and 'super-fractions' (Schmidt) or 'super-particulars' (Robbins) (ἐπιμόρια) of specific numbers.

Ptolemy begins by explaining that the diameter causes the sign and its opposite to meet on a straight line. (See 'AB' in **Figure 1** below.) This is clear enough and also corresponds to the visible sky: if two planets are in opposition, one will be seen to rise at the same time as the other is seen to set.

The next step is to inscribe a semi-circle above the straight line, relating significant numbers to one another. These will first involve the fractions, or *moria*. Bisecting the semi-circle into two right angles (AFC and BFC in **Figure 1**) gives us the square aspect of 90°. One-third of the semi-circle above the diameter produces the side of a hexagon (AD), 60°,

or one-third of the 180° diameter. Doubling the side of a hexagon gives us 120° , which is the triangular interval (AE) that is two-thirds of the 180° diameter.

Figure 1. Aspect Measurements Along the Line and Semi-circle⁹



This gives us all the aspects by degrees that have come down to us as ‘Ptolemaic aspects’, using what appears to be a geometrical method. The *epimoria*, however, give us a different perspective. Let us look at them in relation to the line, rather than the semi-circle.

Ptolemy discusses two ‘superfractions’ or ἐπιμόρια, the ἡμιόλιον (‘half and a whole’) and the ἐπίτριτον (‘a third in addition to a whole’). These correspond to one and a half ($3/2$) and one and a third ($4/3$) respectively. Multiplying the *hēmionion* ($3/2$) by the hexagon (60°) will yield the square (90°). Multiplying the *epitriton* ($4/3$) by the square (90°) will yield the triangle (120°). These proportions are Latinized as the sesquialter and the sesquitercian.

Here is a schematic of fractions and super-fractions on a straight line:

⁹ Diagram from J. Crane, *A Practical Guide to Traditional Astrology* (Reston, VA, 1998/2007), p. 39.

Figure 2. Ptolemy's Superfractions

	Hexagon	Square	Triangle	Opposition
	1/3	1/2	2/3	1/1
0°	60°	90°	120°	180°
	3/2	4/3		

Having mentioned these proportions at the beginning of Chapter 14, Ptolemy drops the matter entirely. However, he is nothing if not intentional and I cannot imagine that he would make a random point and just leave it.

In fact, both the *moria* and the *epimoria* he cites relate to ancient music and, in particular, to the diatonic musical scale. The fact that musical pitches correspond to specific number ratios is a discovery attributed to Pythagoras and throughout history has been associated with the teachings of the Pythagoreans. Using a string or a wind musical instrument, a *fundamental* pitch arises from a vibrating string or a vibrating column of air. Other tones relating to this fundamental pitch can be produced by stopping the vibrating string or air column somewhere up or down its length. If you pluck a string, you will obtain the fundamental pitch which it produces. If you then press down (or 'stop') the string at a point halfway along its length and pluck the resulting half-length of string, you will obtain another pitch one octave higher. This is the same relative tone at a higher pitch, e.g., the interval from C to c. These two pitches are *homophonic*.

The beginning tone, the fundamental, has a ratio of 1:1. A note one octave higher will have a proportion of 2:1. This interval is the *diapason*. If you stop the string at a point half again from the first stopping place, and pluck the resultant one-quarter length of string, the pitch sounded will be two octaves above the fundamental, yielding a proportion of 4:1. Moving through many octaves, the ratios for intervals yield successive multiples of 2.

If you divide the string into thirds and pluck the smaller segment, you will get a pitch between the first and second octaves. If you drop that pitch one octave you arrive at the musical interval of the fifth, which has a ratio to the fundamental of 3:2 (or the distance between C to the G above it) This interval is the *diapente*. These pitches are not homophonic but *consonant*.

If you take the original string or air column and lengthen it by half its length, you will get a pitch that is a fifth lower than the fundamental. If this lower pitch is raised by an octave, you will obtain the musical interval of the fourth above the fundamental, which gives a ratio of 4:3. Using the fundamental C, we arrive at F, which is the distance between C and the F above it. This is the *diatesseron*. This interval is also harmonious.

With C as the fundamental, this yields the fixed pitches of C – F – G – c. The octave or *diapason* is from C to c, or, in the key of G, from G to g. The fifth or *diapente* is the interval from C to G and from F to the c above it. The fourth or *diatesseron* is the interval from C to F and from G to the c above it.

From these fixed ratios other notes were derived, giving rise to the diatonic scale of seven pitches. However, the construction of the diatonic scale in ancient Greek music varied according to the seven ‘modes’, e.g. Lydian, Phrygian, Dorian, and so on.¹⁰

Ptolemy’s use of proportional segments (halves and thirds) of the line for the 180° opposition yields respectively the square (90°) and sextile (60°). The homophonic interval of the octave encloses the eight notes of the diatonic scale. This scale traditionally consisted of two tetrachords: two intervals of the fourth between the lower and upper notes, and a tone in between both tetrachords.¹¹ In the key of C, one tetrachord is between C and F, and the second is between G and c, with a tone remaining

¹⁰ The ‘species of the octave’ (or ‘modes’) preserved the fifth or *diapente* and fourth or *diatesseron* in all the modes except the Hypolydian and Mixolydian, which have a diminished *diapente* and an augmented *diatesseron*. See R. P. Winningham-Ingram, ‘Ancient Greece’ (under ‘Greece’), in S. Sadie, ed., *The New Grove Dictionary of Music and Musicians* (London, 1980), p. 665. The melodies derived from these modes informed the practice of music, and theoreticians have tried to characterize them in terms of their effect on an audience. Plato, in *Republic* (Book III, 398c-399d), criticised some modes and banished them from the ideal state; Aristotle gave a more tolerant description of their effects (*Politics*, Book VIII, Chapter 5, 1339-1342). If adhering to the general principles of harmony can make a well-proportioned soul, the different musical modes seem to conform to discrete personality styles.

¹¹ The Greater Perfect System, which was dominant in ancient harmonic theory, covered two full octaves. Tetrachords could also be found in chromatic and enharmonic forms, although our interest here is in the diatonic. See R. P. Winningham-Ingram, as cited above.

between F and G (called 'emmelic'). In his account of aspects, Ptolemy uses the musical ratios that yield the octave, fifth and fourth, which supply the rudiments of a musical scale, to account for astrology's aspects. Let us now explore why he would choose to do so.

Ptolemy's *Harmonics*

We find similar material in another work by Ptolemy, the *Harmonics*,¹² which is probably earlier than the *Tetrabiblos*.¹³ Although Ptolemy is well-known for his astronomy and astrology, he is also one of the ancient sources for theories of harmony. *Harmonics* deals with a universal field of knowledge for which music and geometry are considered subsets. Much of the material in *Harmonics* is exclusively concerned with music, including setting up exact number ratios for the different modes. Book III of the *Harmonics* applies intervals, scales, and ratios to ethics and psychology, and to astronomy and astrology. Unfortunately, some of this book has been lost to us.¹⁴

In Chapter 9 of Book III, Ptolemy presents the astrological aspects and their ratios to one another. In this presentation, curiously, Ptolemy does *not* use degree numbers for the aspects. Instead, he considers the twelve zodiacal signs as discrete units, and that the whole number relationships between them yield *moria* and *epimoria*. This discussion might have fit better into the *Tetrabiblos* Book I, in the context of his treatment of the qualities and relationships between whole signs.

Ptolemy does not begin this discussion with a line, but rather with the complete circle measuring out twelve units for the twelve signs of the zodiac. I have provided a diagram:¹⁵

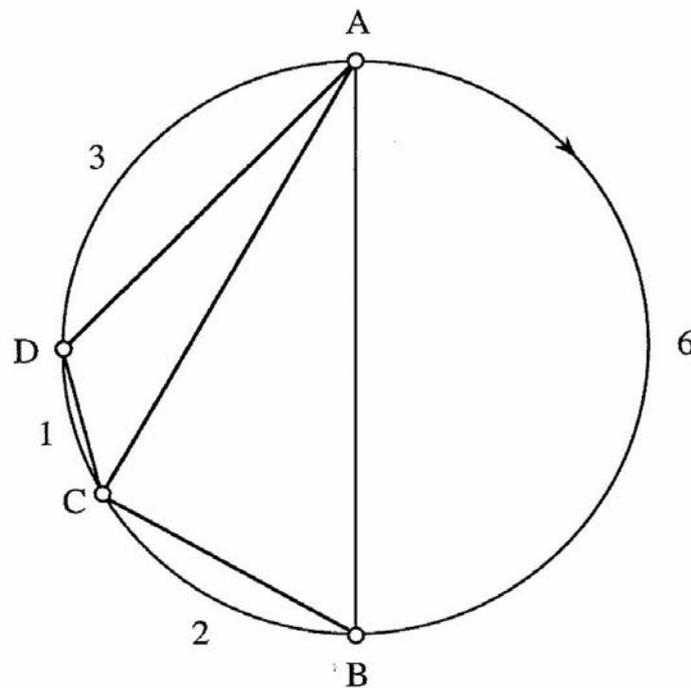
¹² Jon Solomon, *Ptolemy Harmonics: Translation and Commentary* (Leiden/Boston, 2000) [hereafter Solomon, *Ptolemy Harmonics*].

¹³ See N. M. Swerdlow, 'Ptolemy's *Harmonics* and the "Tones of the Universe" in the *Canobic Inscription*', in Charles Burnett, Jan P. Hogendijk, Kim Plofker, Michio Yano, eds, *Studies in the History of the Exact Sciences in Honour of David Pingree* (Islamic Philosophy Theology and Science 54, Leiden/Boston, 2004), pp. 137-80.

¹⁴ See Solomon, *Ptolemy Harmonics*, p. xxx.

¹⁵ Taken from Swerdlow, 'Ptolemy's *Harmonics*', p. 154 (**Figure 2**).

Figure 3. Aspect Ratios from *Harmonics*, Book III



Within this circle, AA is a beginning and end of the zodiac, and hence has the number 12 for the twelve zodiac signs. AB represents the diameter line and six of the signs of the zodiac (the opposition); AC represents four signs and one third of the circle; AD represents three signs and one quarter of the circle. This yields the diameter, triangle, and square of the circle (the hexagon, BC, is not relevant for our purposes here). Ptolemy proceeds to give many permutations of these numbers that yield the same ratios described in *Tetrabiblos* I, 14. I will give the proportions most relevant to our discussion.

For three places together, ABD yields 9, ABC yields 8, and ADC yields 4.

Ptolemy notes the distances that are double from one another (AB doubles AD, and the whole circle doubles AB), which give us the octave

or *diapason*. He also notes the distances that give the fifth and the fourth, the *diapente* and *diatesseron*.

For the 3:2 ratio (the *diapente*), Ptolemy notes that the whole circle (12) is to ABC (8), as ABD (9) is to AB (6), and AB (6) is to AC (4).

For the 4:3 ratio (the *diatesseron*), Ptolemy notes the ratio from the whole circle (12) to ABD (9), ABC (8) to AB (6), and AC (4) to AD (3).

Ptolemy notes that the ratio from AB (the diameter) to AC (the triangle) is 3:2 or the *diapente*, and that the ratio from AC (the triangle) to AD (the square) is 4:3 or the *diatesseron*.

The difference between AD and AC is 1: one-twelfth of the circle. The difference between 4 and 3 spans one zoidion. This would correspond to what was called the *emmelic* interval between the two tetrachords that constitute the diatonic scale in the Greater Perfect System,¹⁶ and is more of a transitional than a concordant interval according to that system. It is important to note that one cannot combine or subtract these segments of the circle to form five units or give a 5:12 ratio. This *would* be discordant, *ekmelic*, and conform to the astrological aspect of the quincunx.¹⁷

We shall now place Ptolemy's account into a larger context.

Harmonic Scales and the Soul of the World

Plato's *Timaeus*, concerned on its surface with cosmology and natural philosophy, is considered to be the most overtly 'Pythagorean' dialogue in the Platonic corpus. In a famous passage on the construction of the world, Plato (or Timaeus) depicts the Demiurge constructing the soul of the cosmos, within which time, motion and form could be realized sensibly, and matter could have a measure of intelligibility (35b-36b). The cosmos' soul will take the form of two bands constituting mixtures of Same and Difference that will eventually form the celestial sphere surrounding the earth.

Prior to this, the Demiurge has to put together the material of the world's soul and then sort it out according to specific quantities that conform to universal ratios. He begins by arranging two series of numbers.

One series uses the multiples of 2 to arrive at 1 – 2 – 4 – 8, etc. The other uses multiples of 3 to arrive at 1 – 3 – 9 – 27, etc. These numbers

¹⁶ See p. 219 and note 11.

¹⁷ Planets five signs from each other were said to be 'averse', having no consonant relationship; indeed, no relationship at all.

can continue indefinitely. The Demiurge then fills the intervals between them, using *arithmetical and harmonic means*.¹⁸

The arithmetical and harmonic means from 1 to 2 are: $1 - 4/3 - 3/2 - 2$. This corresponds to the skeleton of the diatonic scale and the model Ptolemy uses to account for the astrological aspects.

However, by continuing exponential progressions indefinitely and including multiples and means related to the numbers 2 and 3, Plato expands his harmonics further than the realm of ordinary music.

Here we need to explain more thoroughly the arithmetic and harmonic means that Plato employs.

The *arithmetic mean* is the midpoint between two numbers. Therefore the distance between the mean and the greater number is the same as the distance between the mean and the lesser number. We all learned this in primary school and we still know how to do this calculation by adding together the numbers of the two extremes and dividing them by two. This gives us $3/2$ between 1 and 2, 2 between 1 and 3, and 6 between 3 and 9. There are no surprises here.

The *harmonic mean* is a more complex calculation and is more difficult to grasp. The harmonic mean exceeds the lower number *by the same fraction* as the mean is less than the greater number. Thus:

- Between 1 and 2, $4/3$ exceeds 1, the lower number, by $1/3$.
- This same fraction, $4/3$, is less than the number 2 by $1/3$ of 2 (converting 2 to $6/3$); in other words, $6/3 - 2/3 = 4/3$.

We see the same pattern between 4 and 8, where $5 \frac{1}{3}$ exceeds 4 by $1/3$ of 4. $5 \frac{1}{3}$ is less than 8 by $1/3$ of 8. There are two ways to compute the harmonic mean between two numbers. One is the following formula, with A and B being the quantities of the two number at the extremes. This will work perfectly to find the harmonic mean between any two numbers you choose.

$$\frac{2 AB}{A + B}$$

The other way uses the multiples of 2 and 3 that Plato uses, converting the relevant numbers into thirds and halves (multiples of 2 are converted

¹⁸ A fuller explanation is in D. Zehl, *Plato's Timaeus* (Indianapolis, 2000), pp. xl-xli, 20-21; and in Cornford, *Plato's Cosmology*, pp. 66-72.

to thirds, and multiples of 3 to halves). One adds the lower number to its converted numerator, and subtracts the higher number from its converted numerator; the resulting harmonic mean is expressed as a fraction of halves or thirds.¹⁹ This method shows the interdependence between the multiples and the divisions of 2 and 3, though we should note that this second method breaks down when attempting to find the harmonic means between multiples of 5, 7, and so on.

Let us return to Plato's Demiurge. He combines the means between multiples of two and three into a single band. The series of the multiples of 2 and 3 are as follows.

$$1 - 4/3 - 3/2 - 2 - 8/3 - 3 - 4 - 16/3 - 6 - 8,$$

$$1 - 3/2 - 2 - 3 - 9/2 - 6 - 9 - 27/2 - 18 - 27$$

They will make:

$$1 - 4/3 - 3/2 - 2 - 8/3 - 3 - 4 - 9/2 - 16/3 - 6 - 8 - 9 - 27/2 - 18 - 27$$

And so on.

Plato's Demiurge fills the numbers in between by units of 9/8, corresponding to single tones in music. The remaining amounts would be filled in by units of 256/243, which are the semi-tones as represented in Greek musical theory.

¹⁹ I will give one example using multiples of 2 and another using multiple of 3. Between 4 and 8:

- convert the former to 12/3 and the latter to 24/3. We are converting multiples of 2 to fractions with 3 as the denominator.
- Add the lower whole number 4 to 12 (the numerator of the lower number) and you get 16.
- Subtract the higher number 8 from 24, the numerator of the higher number, and you get 16.
- Therefore the harmonic interval between 4 and 8 is 16/3.

Between 1 and 3:

- Convert 1 to 2/2 and 3 to 6/2.
- The lower whole number (1) plus its numerator (2) is 3.
- The higher whole number (3) from its numerator (6) is also 3.
- This will give us 3/2.

Then, having divided the main substance according to these proportions, the Demiurge fashions a very large circular band that he then cuts lengthwise into two and fashions into the form of a Chi. One becomes the Circle of the Same, our celestial equator, upon which the fixed stars move and which moves from east to west – the diurnal cycle. The other becomes the Circle of the Other – the ecliptic. This circle moves from west to east and divides itself further so that the seven planetary bodies can move along it. Thus ordered time becomes possible.

Plato's proportions show intimate relationships between number, music, and the soul of the world. All this seems necessary to explain how the world becomes intelligible, how true opinion could arise, and how true knowledge may be found reflected in the world.

This cosmological story brings us to the motif of the harmony of the planetary spheres, an idea that was pervasive in the Hellenic and Hellenistic worlds and lasted well into the Renaissance. Johannes Kepler's 1619 work *Harmonice Mundi* was probably the last full attempt to bring together the motions of the planets and harmonic ratios, and use Ptolemy's *Harmonics* as a resource.²⁰

Returning to astrology's aspects, we can now use a sequence of the ratios that we see in the *Timaeus*:

$$1 - 4/3 - 3/2 - 2$$

These ratios correspond with the astrological aspects according to Ptolemy's exposition in *Tetrabiblos* I, 14. If 1:1 is the fundamental, 2:1 yields the opposition, 4:3 is the ratio of the triangle to the square and 3:2 is the ratio of the square to the hexagon. Because Ptolemy uses specific numbers in I, 14 for the astrological aspects, he expands the possibilities for aspects beyond those that are between whole signs. By employing the multiples and means that Plato uses, the astrologer finds a wide range of new possibilities.²¹

²⁰ See Swerdlow, 'Ptolemy's *Harmonics*, pp. 137-38.

²¹ There are interesting implications for modern astrologers. Taking the distance between two different numbers and superimposing that on the first 180° of the zodiac gives more intriguing possibilities.

If one uses the numbers between 1 and 3, for example, you get something we haven't seen before and that Ptolemy would not recognize. If 1 and 3 are the extremes, how would the values of the arithmetical and harmonic means become astrological aspects? Using the example $1 - 3/2 - 2 - 3$, the proportion $3/2$

Conclusions

It is clear, from Ptolemy's digression in *Tetrabiblos* I, 14, that using degree numbers makes it possible for astrological aspects to imitate universal laws of harmony and thus account for their effects. Ptolemy presents a correspondence between aspects and musical harmony that allows us to see astrological 'action at a distance' in a new and profound way.

How does harmonic theory help us account for the aspects of astrology? Harmonious pitches (homophonous or consonant) can be said to meet each other, to interact with each other. In music they act upon each other *because* of their distance along the scale. I know of no other phenomenon in nature in which interaction is based upon number ratios related to distance between two agents.

The contrasting experience is also quite familiar – the discordant and ugly result of striking the *wrong* note. Within the context of tonal music, this is the result of producing an unmelodic interval in the context of the harmonics established within the piece being played. An accomplished musician or composer may find a way to resolve the discord, but does so by finding a way back to the original harmonic intervals. Unmelodic intervals may correspond to disharmony in the world, in the individual soul, and between planets affiliated by dispositorship but in disconnected zodiacal signs.

In addition, two or more harmonious pitches played together also create a blend of sameness and difference that is analogous to the relationship between a visual perceiver and its object of perception. Musical pitches and intervals can be represented by numbers and ratios. Their arithmetical properties allow us to move from the aural sense perception of musical tones to an analogous intellectual concept of

corresponds not to the sextile but the *semi-square*, an astrological aspect of 45° , half the square of 90° . This aspect requires degree numbers, not whole signs. The semi-square violates Ptolemy's use of whole masculine and feminine signs to describe the effects of different aspects.

Using the extremes of 1 and 9 gives $1 - 3/2 - 2 - 3 - 9/2 - 6 - 9$.

The proportion $3/2$ is the semi-sextile of 30° ; 2 yields the aspect of 40° which astrologers know as the *novile*, dividing the 360° circle into ninths. This is the foundation of the modern Ninth Harmonic: 3 is the sextile, $9/2$ is the square, and 6 is the trine. Modern astrologers who use the Ninth Harmonic can derive some justification of their methods from these proportions.

harmony. This harmony may manifest itself in the soul of the world, or in the soul of the individual, and account for the aspects of astrology.²²

The logical model for principles of harmonics is music, not geometry, since the parts of a geometrical figure do not interact with each other based upon the ratios of their distances. Because divisions and multiples of 5 or 7 do not fit into these harmonic models, they cannot themselves form the basis for either musical harmonies or astrological aspects, if the correspondence between aspects and harmonic intervals is to be taken seriously.

Ptolemy's argument in the first part of Chapter 14 is indeed a digression. The remainder of *Tetrabiblos* I uses the natural philosophy of his day to account for astrological effects in general. His argument for aspects, however, derives from Pythagorean and Platonic sources as evidenced in his earlier *Harmonics*. Yet his digression roots us in some of the basic principles of the western intellectual tradition, and gives us the possibility that the symbols of astrology have something to do with the nature of reality.

²² The topic of the harmony of the soul is beyond the scope of this paper. It is significant that Plato's *Timaeus* is supposed to have taken place the morning after the long discussion of the 'just' – well proportioned – soul in the *Republic*. (Also see E. McClain, *The Pythagorean Plato* (York Beach, ME, 1978). In Ptolemy's *Harmonics* III, 5, he compares the harmonious activity of the soul as the integration of its parts to the familiar intervals of the *diapason*, *diapente*, and *diatesseron*.