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Translations of Kepler's Astrological Writings

Part III, Section 4. Kepler on Aspects, 1602

Translated by Cornelia Linde and Dorian Greenbaum; annotated by Dorian Greenbaum

Taken from Opera Omnia *1, pp. 309-310, 322;* Gesammelte Werke *14, pp. 268-270;* GW *14, pp. 254, 331-332.*

Text from GW *not appearing in* OO *is written between forward slashes (/). However, the version in* GW *which corresponds to* OO *1, pp. 309-310 is filled with lacunae and appears to be quite corrupt. In the notes, we have noted instances of difference between* GW *and* OO*, especially when* OO *contains a more complete, or more understandable, text.*

The first part of this discussion of aspects is excerpted from a letter Kepler wrote to David Fabricius on 1 October 1602. This is the same letter in which Kepler responded to Fabricius's questions on his life events (see ***Part I.1.2*** *in this volume). The second excerpt is from Kepler's response to Fabricius's question about aspects, part of the same set of letters (1-5 August 1602 and 2 December 1602) in which Fabricius and Kepler discussed astrological theory (see* ***Part III.2*** *in this volume).*[1]

These excerpts provide a window into Kepler's evolving views on aspects. Written in 1602, they deal with the geometry, harmonics and philosophy of aspects, topics that had occupied Kepler's mind for years (at least since the writing of Mysterium Cosmographicum, chapter 12

[1] Because of the informal letter format, and because of lacunae in the texts, translating this section was very difficult. In the following commentaries on the text, we are grateful to Dirk Grupe, whose assistance with mathematical terminology and explanation of geometrical concepts has been extremely helpful.

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[published in 1597]) and finally culminated in his 1619 book, Harmonices Mundi Libri V *(*Five Books on the Harmonics of the World*).*[2] *In these excerpts we see a middle stage of development in Kepler's thinking on the rationale behind the 'stellar rays' (i.e., aspects).*[3]

In these passages, Kepler's interest in the theory behind astrological aspects leads him to base their effectiveness on principles in geometry, especially empoying the concept of rational and irrational polygons. He looks at both standard and non-standard astrological aspects (formed from parts of a circle or the sides of certain polygons). In the course of his exposition of the theory behind aspects, he explains that the astrological conjunction is equivalent to the whole circle or point and the opposition is equivalent to the diameter. He also discusses the sextile (formed from the side of a hexagon), the square (the side of a square) and the trine (the side of a triangle) These five are the standard Ptolemaic aspects. In addition, Kepler discusses the polygons from which derive the non-standard aspects (which he developed in his weather forecasting work): the pentagon (yielding the quintile and bi-quintile), decagon and octagon (this last related, in Kepler's demonstration, to the sesquiquadrate).The points made in this dicussion of aspects are further developed in Harmonices mundi*, Book IV, Propositions 9-14.*

Kepler prefaces his remarks on the efficacy of certain astrological aspects with a discussion of rational and irrational polygons.[4] *He*

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[2] Johannes Kepler, *Harmonices mundi libri V* (Linz, 1619).

[3] Correspondents to whom Kepler wrote on this topic in the early days included Herwart von Hohenburg, Edmund Bruce and Michael Maestlin (all in 1599). See J. V. Field, 'A Lutheran Astrologer: Johannes Kepler', *Archive for History of Exact Sciences* (1984), vol. 31/3, pp. 204-205.

[4] We are using 'rational' and 'irrational' in the way that Kepler meant them, which is slightly different from their modern conception (here I quote from ADF, p. 21 n. 17): 'In modern usage "rational" denotes numbers expressible in the form $\frac{a}{b}$, where *a* and *b* are integers. Euclid, and Kepler and his contemporaries, use "rational" (and its equivalents) to cover not only these numbers but also numbers whose squares are expressible in this form. Thus the meaning Kepler says arithmeticians give to "surd" (lit. "deaf") is equivalent to the modern "irrational".' To complicate matters further, by the time he wrote *Harmonices mundi*, Kepler preferred 'expressible' (*effabilis*) as a translation for ῥητός, not 'rational': 'It [the square of a line] is said to be ῥητή δυνάμει, "Expressible in

considers that some degree of rationality is a criterion for effectiveness. It is important for him to demonstrate rational proportions between the radius or diameter of a circle and the side-lengths of inscribed polygons, either their base values or their squares (i.e. first or second powers). He uses not only the side-lengths themselves, but also squares formed from the side-lengths of polygons to make a comparison with squares on the diameter or radius, in order to show if their proportions are rational or irrational. Kepler demonstrates that the side of the hexagon is rational in itself, because its side is the radius, i.e. a 1:1 proportion, which is rational. On the other hand, the inscribed square's side has an irrational proportion to the circle's radius (the square root of 2 is irrational). But if its side is squared, it does come into rational proportion to the radius, and thus the square is rational. The same can be done with the triangle, whose squared sides are in rational proportion to the squared radius. (The hexagons, squares and triangles, of course, create the sextile, square and trine aspects.)

The squares on the sides of the pentagon (whose angles lead to the quintile and bi-quintile), however, do not make a direct rational proportion to the square on the diameter, so this puts them in an inferior position (Kepler's words: loco posteriore*) to the rational hexagons, squares and triangles. However, using a subtractive method, where certain quantities can only be produced, and their relation to others be expressed, by subtraction, Kepler finds a way to compare the sides of the pentagon to the radius. This subtractive procedure, which he describes as 'intellectual' (*a mente*) rather than physical, allows polygonal sides which would not otherwise be comparable to be compared to the diameter or radius, and so demonstrates their value as aspects. These two methods of comparison, one using the quantitative existence of objects and the other their inter-relatedness through intellectual means, will be further explored in* Harmonices mundi *(see note 17 below).*

In addition his geometrical demonstrations, Kepler draws on harmonics in music theory to augment his argument that the quintile, bi-quintile and sesquiquadrate can be put into the category of working aspects.

The first excerpt begins with Kepler's preliminary discussion of rational and irrational polygons. This purely mathematical demonstration then allows Kepler to make his arguments for the aspects later

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square" (*effabilis potentiâ*)' (*Harmonices mundi*, Book I, Definition XIV; translation in ADF, p. 21).

discussed. Though the argumentation is dense, we include it to show how serious Kepler was about finding a rational justification for the efficacy of aspects.

[*OO* 1, p. 309; *GW* 14, pp. 268.197-269.227] (Kepler's letter of 1 October 1602)
I think I see these aspects as acting more strongly, but their power has to be appraised by the method (*ratio*) of demonstration: the side of a hexagon itself is rational (ῥητόν)[5] *per se*, for it is equal to the radius; the square[6] on the side of the square is rational, for it is half of the square on the diameter; the square on the side of a triangle is to the square on the diameter as 3 to 4. The opposition itself is the diameter. The conjunction is pure identity, or monophony (μονοφώνια), or in geometry a whole circle or a point, or the nullity of an angle of rays, i.e. coincidence.[7] Now the squares on the quintile and biquintile are connected to the square on the diameter as 5 to 4. Not even [their squares] have a rational proportion to each other or to the squared diameter, as a number to a number,[8] and so on this account they are in an inferior position. See Adrianus Romanus's *Method of Polygons*.[9] For neither the first nor the second, but only their third squares (*quadrata*) are expressed in [whole] numbers. The power (*potentia*) of the pentagonal side is comparable to (*aequalis est*) the power of the radius and the power of the decagonal side, because it has its determination in a remote degree.[10] /For the squares, expressed through a

[5] Though by 1619 Kepler preferred 'expressible' for ῥητός, we shall use the term 'rational' to translate it in this 1602 text, with the caveat that Kepler's definition was not the modern one (see note 4 above). For the meaning of ῥητόν as 'rational' in a mathematical sense, see LSJ, s.v. ῥητός.

[6] 'quadratum' in *OO*; ellipsis in *GW*.

[7] The order of sentences in this section is different in *OO* and *GW*; we follow *GW* here.

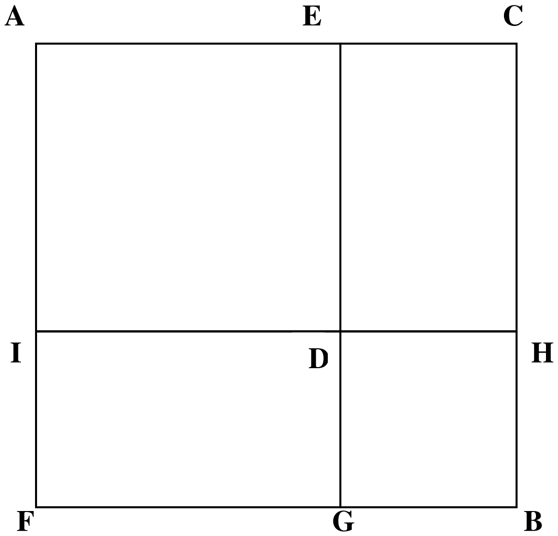
[8] As Aiton, Duncan and Field note, 'Kepler uses *numerus* ("number") in the sense of the Greek ἀριθμός, to mean a positive integer' (ADF, p. 58 n. 194).

[9] Adriaan van Roomen (1561-1615), Flemish mathematician. He and Kepler met in 1600 in Prague. The *Method of Polygons* was published in 1593.

[10] Kepler uses the same phrase, 'remote degree', in *Harmonices mundi*, Book III, Proposition 3, to describe how 'strings…in more distant proportion are in consonance at a more remote degree.' (Translation in ADF, p. 153.) 'Degrees of

number, may add up to a sum, the square of the radius and a half …; … it is not a square [number?] (*quadratus*), but nevertheless since it yields .. of the diameter by a geometer./[11]

Figure 1. Kepler's Illustration of Rational and Irrational Polygons, from his Letter to Fabricius of 1 October 1602[12]



knowledge' according to degrees of irrationality were introduced by Kepler in Book I (see ADF, p. xxi; also the translation on p. 19, Definition VIII: 'A quantity is said to be knowable if it is either itself immediately measurable by the diameter, if is [*sic*] a line; or by its [the diameter's] square if a surface: or the quantity in question is at least formed from quantities such that by some definite geometrical connection, in some series [of operations] however long, they at last depend on the diameter or its square.'). For Kepler, 'knowability', along with congruence, was one of the requirements for the influence of aspects. Interestingly, Kepler's first mention of the importance of 'knowability' (*scibilitas*) was earlier in this 1 October 1602 letter to Fabricius (*GW* 14, p. 266: this observation in ADF, p. xxi and n. 48).

[11] Lacunae here make a sure translation difficult: 'Nam Quadratum semidiametri et dimidium … quadrati faciant summam numero expressi … quadratus non est, sed tamen quia . . diametri potest a Geometra.'

[12] This drawing is copied from the one that appears in *OO* 1, p. 309. The illustration in *GW* 14, p. 269, shows the diagram as forming only squares, not some squares and some rectangles. The letters used in Figure 1 combine those of *OO* and *GW*, to create the proper alphabetic sequence from A to I.

Now Kepler explains his diagram:

Let that square be AB, whose side AC already **[*GW* 14, p. 269]** is incapable of being expressed by a number [*i.e. rational*]. But for this square [AB], whose number is known, another [smaller] square AD is taken away, which itself with its side can be expressed by a number. Therefore AC is cut in E by a known proportion; but EC can still not be expressed by a number, since the rational length AE subtracted from the irrational[13] AC leaves the irrational EC. And because the irrational [side-length] EC makes DB, itself derived in it,[14] therefore DB is irrational. And from this other rational square, the square of the radius, the square on the side of the pentagon is composed. This square [DB] is not *per se* comparable to the diameter, but [has to be determined] by a subtractive procedure (*per ablationem*). For AB contains more than AD and DB, namely FD and DC, and it is comparable [*i.e. rational*]; in the same way AD is comparable [*rational*]. That is understood; at the same time, however, FD and DC[15] turn out to be incomparable [*i.e., irrational*], yet they retain their determination from comparable [*i.e., rational*] AB [and] AD, although the sought for DB is left incomparable [*i.e., irrational*].

Finished with his geometrical demonstration, Kepler now moves to the heart of his arguments about the efficacy of astrological aspects formed from the sides of polygons.

[*OO* 1, p. 309; *GW* 14, p. 269.227] Therefore this, as well, is the marvelous nature of quantities, that they are determined[16] by a subtractive procedure (*ab ablatione*). Although let me add that this is the whole manner of being (*essentiae ratio*). So in one way they exist (in

[13] Here Kepler has used the Greek word ἄρρητῳ, lit. 'unspeakable' or 'unutterable' (the text incorrectly accents the word thus: ἀῤῥητῳ). In mathematics, τὰ ἄρρητα are irrational numbers (LSJ s.v. ἄρρητος IV.) This word is the antonym of ῥητός, which Kepler will eventually translate as *effabilis*, 'expressible' (see note 4 above).

[14] *OO* has 'Et quia EC, ἀῤῥητον [sic], in se ipsum ductum facit DB...'; *GW* has 'Et qua E C ἀῤῥητον [sic] in seipsum…facit DB...'. We follow *OO* here.

[15] In *GW*, 'IG, FG'. (But FG is a line in both drawings.)

[16] Lit. 'obtaining its determination' (*sortiens suam determinationem*).

quantitative existence) but they are put in comparison in another way. This occurs by intellectual conception (*a mente*).[17] But to the matter at hand.

The following paragraph gives Kepler's reasoning for using two methods (quantitative existence and mental conception) to show how some irrational aspects can be valid. So he describes both a quantitative existence of objects as well as their inter-relatedness by the intellectual manipulation of subtraction. He then uses the relationship of polygonal sides to certain musical harmonies to argue that the parallel with music also gives value to the aspect. This allows him to justify the value of the irrational pentagon, decagon and sesquiquadrate. In addition, the pentagon and the aspects derived from it have a relationship to the golden section (see note 25 below), which also promotes their value.

This [particular] example (*genus*), the side of a pentagon [*i.e. that which creates a quintile*] stands in remote comparison (*in remota comparatione*)[18] and is not in accord (*abhorret*)[19] with the rest [*i.e. with polygonal sides which can be compared to the diameter or radius without subtraction*]. This disturbed me greatly. For it seemed in this scenario that nothing could have value, if it is neither understood by itself [*i.e., the side of a hexagon can be understood as rational in itself because it is equal to the radius*] nor can it be compared [directly]. But music encouraged me.[20] For although the minor third (*tertia mollis*) derives

[17] In 1619, Kepler will expand on the idea of the knowledge of quantity through the intellect in *Harmonices mundi*, Book III; see, e.g., '...since the terms of the consonant intervals are continuous quantities, the causes which set them apart from the discords must also be sought…and since it is Mind which shaped human intellects in such a way that they would delight in such an interval (which is the true definition of consonance and discordance) the differences between one and the other…should also have a mental and intellectual essence…' (translation in ADF, Book III, Introduction, p. 139).

[18] I.e., remote because it can only be compared by using the subtractive procedure, not directly.

[19] Lit. 'shrink from'.

[20] Kepler also discusses harmonic ratios, music and aspects in *De stella nova*, Chapter 9 (see Patrick Boner's translation in this volume, pp. 229-231). Up until 1608, Kepler believed that aspects and musical harmonies had the same origin and exactly corresponded, but he then modified his view, considering similarities

from the [*rational*] hexagon[21] and the major third (*tertia dura*) from the [*irrational*] pentagon,[22] the major third is certainly more pleasant (being major), while the minor is a little bit off so that it does not harmonise. Therefore I had to come up with a reason by which many other figures would also make consonances (*consonantia*), although they are quite irrational (ἀῤῥητοι [sic]), unless they are prevented by other causes. So you see in the earlier letter that a decagon, 3/10 [*tri-decagon*], one octave etc., and many others, in fact infinite others, were abandoned. /…the difference being consistent with the theory (*ratio*) of equality: then all the irrational [aspects] fall except the quintile, bi-quintile and sesquiquadrate [» ☆ and #]./ The cause of why, in the pentagon,[23] the major third is more powerful than the minor, can also be demonstrated geometrically. **[*OO* 1, p. 310]** For although the quintile and the biquintile are certainly in an inferior (*posterior*) position among the demonstrative figures, yet at the same time, through this retreat and this flight from equatability (*aequatio*) [*i.e., comparability*], they come into[24] a divine proportion among all the geometrical shapes.[25] Therefore even if they lack

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between consonances in music and aspects, but not absolute equivalence. (See ADF, p. xxxi and n. 88 and *Harmonices mundi*, Book IV, Chapter 6 [ADF, p. 351]).

[21] The minor third has a ratio of 6:5.

[22] The major third has a ratio of 5:4.

[23] *OO* has 'sexangulo', but there is a lacuna in *GW*; we have chosen to use 'quinquangulo' ('pentagon') because earlier Kepler associated the major third with the pentagon (a ratio of 5:4), and the sentence following this one deals with the quintile and bi-quintile.

[24] *OO* has 'perveniatur'; *GW* has 'perveniant'. We use *GW* here.

[25] This 'divine proportion' is the golden section, or golden ratio, which is the division of 'a line segment such that the ratio of the large part to the whole is equal to the ratio of the small part to the large part'. (Euclid, *Elements*, Book VI, definition 3, quoted in Charles F. Linn, *The Golden Mean: Mathematics and the Fine Arts* [New York, 1974], p. 20.) It can also be expressed as 1.618... and written in modern notation as φ, after the Greek sculptor Phidias, who used it in his sculptures. Luca Pacioli's 1509 book *De divina proportione* was illustrated by Leonardo da Vinci with drawings of the five Platonic solids. The proportion between the sides of a pentagon and its diagonals are the golden section, and this is what Kepler is referring to when he talks about the quintile and bi-quintile. In

numerability, or the nature of number [*i.e., they are not rational*], on the other hand they have a divine proportion which the ordinary aspects lack.[26] Because of that, and because the pentagon is employed in making the body, and indeed the principal [body], and the 12 biquintile figures are employed in making the most beautiful regular growth (a star), they are to be grouped with aspects.[27] The last of the [geometrical] figures in both designations is the /⋕ or/ sesquiquadrate.[28] For the rest, they are able to achieve little in effect, but more than any other non-harmonic ones, just as in music the minor sixth (*sexta mollis*) is still a consonance (*consonantia*), so that even though it is weak, it does not hurt the ears as those which are discordant [do]. For the perceptible conformity (*convenientia*) of music with my harmonic reasonings makes it certain

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an interesting coincidence, Michael Maestlin was the first to calculate the golden section as a decimal, in a margin note added to a letter Kepler wrote to him at the beginning of October 1597: his calculation is 'about .6180340' (*6180340 fere*) (see *GW* 13, pp. 142 [Kepler's diagram] and 144 [Maestlin's margin note on the diagram]).

[26] The relationship between the sides of the pentagon and its diagonals are in the proportion of the golden section.

[27] Instead of 'they are to be grouped with aspects', *GW* has 'ubi quinque anguli nimium circumstantes uno … plano utuntur', 'Where the five angles on the circumference particularly use a single plane.' In *Harmonices mundi*, Book IV, e.g. Proposition 7 (see ADF, pp. 336-339), Kepler makes a distinction between figures at the circumference and figures at the center. I think that is what Kepler means here, that five angles on the circumference (literally, 'the five angles standing around in a circle') make a figure, i.e., the pentagon, on a single plane.

[28] 135º, a square and a half (90º + 45º). The rest of this paragraph appears riddled with lacunae in *GW*; in the main text we supply the version in *OO*, but provide an attempted translation of *GW* here: '/For it can be [related to the diameter in] two [ways]. …[*The first way*] the square is rational (ῥητον), [taking] of course a quarter (*quarta pars*) of the square's side-length from the radius **[*GW* 14, p. 270]** and a half side-length… Moreover [other rational aspects are] not [in] divine proportion. And in equating the square on the diameter it receives an association not … as … biquintile, but dissonant from a harmonic ratio (*peregrinum a ratione harmonica*), namely the side of an octagon. For that reason also in Music the minor sixth which originates from this [the octagon] is most common, and just about … and broken up. And so it is harmonic neither in effect…/' It seems that in this corrupted text, Kepler is relating the side of the square, on the one hand, and the side of the octagon, on the other, to the sesquiquadrate.

that my reasons must not be denied, wherefore they are also not to be despised in astrology.

This [is all] for now, just to keep you and me engaged.

The second excerpt contains Fabricius's original question about aspects, occurring in a postscript to his August letter on astrological theory. Kepler's response in December 1602 postdates his earlier comments of October 1602, but is important because it includes some additional information on the topic.

[*OO* 1, p. 322; *GW* 14, p. 254]
FABRICIUS (Postscript to letter of 1-5 August 1602, O.S.): [*Pulkova X, 18r*] ~~1.~~ Since contrary to expectation, my earlier letter remained here /for one day and the next,/ I wanted /also/ to add my later thoughts to the earlier ones, /so that the number of questions would be greater and the exercising of the intellect more fruitful on both sides. 1./ It is asked whether the opposition [S] and square [□] of the good [planets] as, for example, Jupiter [y] and Venus [r], are good or bad. I think all aspects of the good [planets] are good, and of the bad, bad. Although the power in both the sextile [G] and trine [F] is less.

[*OO* 1, p. 322; *GW* 14, pp. 331-332]
KEPLER (letter of 2 December 1602): [*Pulkova X, 45r*] /29./ … **[p. 331]** Your postscripts call me back to astrology. [As for] just which aspects are good [and] which are bad: I think that in regard to the aspects, they are not to be perceived astrologically through marks of goodness and badness, but rather through strength and weakness.[29] The one that is stronger is the one whose expression is stronger. First[30] is the bodily conjunction [☌], because **[*GW* 14, p. 332]** here is sameness (*identitas*). Second, the bodily opposition [S] because it is the diameter of the circle, bisecting the circle. Third, the square [□][31] because the square on the side

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[29] Kepler makes the same claim about the strength and weakness of aspects in *De stella nova*, ch. 9, first paragraph (see Patrick J. Boner's translation in this volume, pp. 225-228).

[30] This ordering of aspects from most to least efficacious, and its causes, is developed in more detail in *Harmonices mundi*, Book IV, Chapter 5, Propositions 9-14 (see ADF, pp. 340-347).

[31] *OO* has 'Tertio quadratura'; *GW* has '3. □.'

[of a square] is half of a square on the diameter.[32] Fourth, the sextile [G] and trine [F] because the side of a hexagon is equal to the radius, and the side of a triangle yields ¾ of a diameter.[33] Fifth, the quintile [⋆] and bi-quintile [☆], because the squares of both are connected with the square of the diameter as 5 [is related] to 4 [*i.e., the proportion*].[34] [Margin note: anything incomparable[35] to the diameter is irrational.] Sixth the sesquiquadrate, because it is irrational and without association. On the other hand, the octagon in fact aids it in being comparable to *(ad aequandum*) the square on the diameter, [*Pulkova X, 45v*] but the octagon is not among the harmonic [aspects]. Now, for the planets, there are no good and bad, but hard, soft, hot, cold, wet, dry. The sextile [and] trine, [and] the sesquiquadrate [GF, #], fit with the soft and wet, when the experience of the ears is transferred from music to astrology; the quintile [⋆] and biquintile [☆] are more suited to the hard and the burning.

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[32] 'quia lateris quadratum est quadrati diametri dimidium'. In other words, the square created using the side of the square is half of the square created by using the length of the diameter. See above, p. 306, for Kepler's wording of this idea in a slightly different form (in *OO* 1, p. 309, *GW* 14, p. 268.199-200).

[33] *OO* has: 'Quarto sextilis et trigonus, quia latus sexanguli aequale est semidiametro, et latus trianguli potest ¾ diametri'. *GW* has: '4. Get F quia latus G potest ¼, F ¾ diametri: '...because the side of a sextile yields ¼ of the diameter [squared, and] the [side of a] trine yields ¾ of the diameter [squared]. This re-expresses the thoughts of *OO* 1, p. 309; *GW* 14, p. 268.198-199 and 200-201; see p. 306 above.

[34] *GW* has '5. ⋆☆ quia juncta utriusque quadrata possunt 5/4 diametri.' '5th, the quintile and biquintile, because the squares of both connected [to the square] of the diameter yield [a ratio of] 5/4'. This restates *OO* 1, p. 309; *GW* 14, p. 268.204-205 (see p. 306 above).

[35] Lit., 'anything separate' to the diameter is irrational ('seorsim vero quilibet ad diametrum est irrationalis').